Handout 10
NPV's Place as a Lifetime Aggregator for the Individual Agent

Given various forces impinging on an individual agent who is trying to make efficient current decisions – when these decisions affect current utility as well as future utility – what maximand formulation best captures the agent's dynamic decision making problem? Maximizing utility in a static setting is one thing. Can utility maximization be expanded to adequately model intertemporal decisions?

<u>Three preference-based forces</u>

1. *Time preference (impatience)*: now dollars (or now-utility monetized into $) are worth more today than tomorrow dollars to typical agents. Assuming exponential preferences, a discount rate δ is able to convert future $ into today $ and thereby account for time preference.
2. *Risk*: future outcomes are uncertain and expected value outcomes do not always capture agents' attitudes about uncertain prospects. Risk aversity is the norm. The expected utility model is admittedly imperfect in its ability to mimic particular consumer behaviors (in light of some famous paradoxes), but it does address risk formally and consistently; it is a reasonable and practical approach in many circumstances.
3. *Consumption smoothing*: from a multiperiod perspective, large experienced utilities in some periods are imperfect substitutes for small utilities in other periods. Consumers prefer smooth flows of utility over time. Expressed differently, the consumer has a limited willingness (resistance) to substitute utility across time. [Receiving 100 added utils during a high-util period doesn't compensate for a 100 util loss during a low-util period regardless of the time ordering of these two periods.]

A deft framework for incorporating these elements within a single modeling structure is that of *recursive utility* advanced by Epstein and Zin (1989, 1991). The model nests expected present value (EPV) or expected net present value as a special case, thereby illuminating EPV's standing as a potentially complete (or not) welfare measure. The model has two building blocks. Once they are each outfitted with functional form selections, they produce various aggregates for lifetime utility depending on single parameters representing the 3 forces noted above. The following developments are drawn mainly from Epstein and Zin (1991).

Part A (risk aggregation): Now is period t=0. Future utility, U_t t>0, is random given the present state of knowledge. Presuming the expected utility framework is applicable, future utility in period t is well measured by its certainty equivalent, given by the function $\mu[U_t]$.

Part B (time aggregation): There is a function W that aggregates utility experienced in different periods into lifetime utility. There is sufficient consistency assumed for the agent's preferences that this aggregator function can be expressed recursively:

(1) $U_t=W(c_t, \mu[U_t])$

where U_t is lifetime utility commencing in period t and c_t is the value of consumption in period t. Hence, U_0 is determined recursively, by substituting U_1 in the aggregator's right-hand side which then causes U_2 to appear, and so on.

Functional form for Part B:

(2) $$W(c,z)=\begin{cases}\left[(1-\beta)c^{\rho}+\beta z^{\rho}\right]^{1/\rho} & \text{if } 0\neq\rho<1\\ (1-\beta)\log(c)+\beta\log(z) & \text{if } 0=\rho\end{cases}$$

where the discount factor β is related to the discount rate δ via β=1/(1+δ), and resistance to intertemporal substitution ρ is related to the "intertemporal constant elasticity of substitution" σ via σ=1/(1−ρ) with σ∈[0,∞). Hence, ρ=(1−σ)/σ. Infinite elasticity of substitution occurs for ρ=1.

Functional form for Part A:

(3) $$\begin{cases}\mu[x]=[Ex^{\alpha}]^{1/\alpha} & \text{if } 0\neq\alpha<1\\ \log(\mu)=E\ \log(\mathrm{x}) & \text{if } \alpha=0\end{cases}$$

where α is the constant relative risk aversion parameter with risk aversion that increases as α rises, and E is the usual expectation function. Ex^{α} means $E[x^{\alpha}]$. Risk neutrality occurs when α=1, and α=0 indicates log risk preferences (Knapp and Olson 1996). Howitt et al. call 1−α the "risk aversion coefficient" (2005).

Hence, there are separate parameters (β or δ, ρ or σ, and α) for the three forces noted previously.

Assuming neither α nor ρ is zero, (1)-(3) combine to give the following recursive function.

(4) $$U_t=\left[(1-\beta)c_t^{\rho}+\beta\left(EU_{t+1}^{\alpha}\right)^{\rho/\alpha}\right]^{1/\rho} \quad \text{or} \quad U_t=\left[\frac{\delta}{1+\delta}c_t^{\rho}+\frac{1}{1+\delta}\left(EU_{t+1}^{\alpha}\right)^{\rho/\alpha}\right]^{1/\rho}$$

Repeated substitutions of this function into itself for its 2nd inner term (U_{t+1} and then U_{t+2}, etc.) further resolves the function, but doing so is really only useful for special (nested) cases of the general preference parameters, especially ρ and α.

For the prime example, assuming the special case of ρ=α, the following several algebraic steps expand the first recursive statement of (4) for lifetime utility at t=0.

$$\begin{aligned}U_0^{\alpha}&=(1-\beta)c_0^{\alpha}+\beta EU_{t+1}^{\alpha}\\&=(1-\beta)c_0^{\alpha}+\beta E((1-\beta)c_1^{\alpha}+\beta U_2^{\alpha})\\&=(1-\beta)c_0^{\alpha}+(1-\beta)\beta Ec_1^{\alpha}+(1-\beta)\beta^2EU_2^{\alpha}+\beta^3EU_3^{\alpha}\\&=(1-\beta)\sum_{t=0}^{\infty}\beta^tEc_t^{\alpha}\\&=\frac{\delta}{1+\delta}\sum_{t=0}^{\infty}\frac{Ec^{\alpha}}{(1+\delta)^t}\end{aligned}$$

The positive scalar δ/(1+δ) doesn't contribute and can be dropped. Note then that the right-hand side of the final result would be EPV if not for the appearance of the constant relative risk parameter. The final result is

$$U_0 = \left[\sum_{t=0}^{\infty} \frac{Ec^{\alpha}}{(1+\delta)^t} \right]^{1/\alpha}$$ for this special case (ρ=α).

Again, in a maximization context, it would seem that the 1/α exponent does not matter and can be dropped. The remaining appearance of α shows that we have expected value iff α=1.

Using eqs. (1)-(3) or (4) and specific cases involving ρ or α, various results are obtained and entered in Table 1. Most of these results are still in recursive form.

The two cells enclosed by dashed borders is the special case for which EPV emerges as the lifetime utility aggregator.

References

Epstein, L. G. and S. E. Zin. "Substitution, Risk-Aversion, and the Temporal Behavior of Consumption and Asset Returns - A Theoretical Framework." *Econometrica* 57 (1989): 937-69.

Epstein, Larry G. and Stanley E. Zin. "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: An Empirical Analysis." *Journal of Political Economy* 99 (1991): 263-86, 10.2307/2937681.

Howitt, Richard E., Siwa Msangi, Arnaud Reynaud, and Keith C. Knapp. "Estimating Intertemporal Preferences for Natural Resource Allocation." *American Journal of Agricultural Economics* 87 (2005): 969-83, 10.2307/3697783.

Knapp, K. C. and L. J. Olson. "Dynamic Resource Management: Intertemporal Substitution and Risk Aversion." *American Journal of Agricultural Economics* 78 (1996): 1004-14, 10.2307/1243856.

Table 1. Lifetime Utility: General and Special Cases involving ρ and α

U_0		ρ			
		generally (≠0,1)	**α**	**1**	**0**
α	**generally (≠0)**	$\left[\frac{\delta c_0^\rho + \left(E\,U_1^\alpha\right)^{\rho/\alpha}}{1+\delta}\right]^{1/\rho}$	$\left[\sum_{t=0} \frac{Ec_t^\alpha}{(1+\delta)^t}\right]^{1/\alpha}$	$\frac{\delta c_0 + \left(E\,U_1^\alpha\right)^{1/\alpha}}{1+\delta}$	$\frac{\log[c_0^\delta (EU_1^\alpha)^{1/\alpha}]}{1+\delta}$
	1	$\left[\frac{\delta c_0^\rho + \left(E\,U_1\right)^{\rho}}{1+\delta}\right]^{1/\rho}$	$\sum_{t=0} \frac{Ec_t}{(1+\delta)^t}$ (i.e. Expected PV)		$\frac{\log[c_0^\delta EU_1]}{1+\delta}$
	0	$\left[\frac{\delta c_0^\rho + \exp[\left(E\log U_1\right)^{\rho}]}{1+\delta}\right]^{1/\rho}$	$\frac{\delta \log c_0 + E\log U_1}{1+\delta}$	$\frac{\delta c_0 + \exp[E\log U_1]}{1+\delta}$	$\frac{\delta \log c_0 + E\log U_1}{1+\delta}$

α≤1 is the constant relative risk aversion parameter; α=1 represents risk neutrality.

ρ≤1 is resistance to intertemporal substitution; ρ=1 represents the absence of resistance.

δ is the ordinary discount rate. E is the expected value operator. Ey^k is $E[y^k]$.