Handout 4

Expanding the Arrow-Debreu economy to include a nonrival good

(starting directly from Handout 1 after adding modifications 1-4 below)

1. Introduce good z which is nonrival in consumption
2. but is rival in production z^j
3. ω_z is society's initial endowment of z: $\omega_z = \sum_{h=1}^{H} \omega_z^h$
4. z^h is the amount of z that h chooses to buy, but all consumers enjoy all of z

Efficiency

From HO 1, part E:

$$L = \sum_h \lambda^h \left(U^h\left(x^h, z\right) - \overline{U}^h \right) - \sum_j \alpha^j f^j\left(y^j, z^j\right) + \sum_i \beta_i \left(\omega_i + \sum_j y_i^j - \sum_h x_i^h \right) + \beta_z \left(\omega_z + \sum_j z^j - z \right)$$

Still get FOC's [3] & [4] pertaining to y_i^j & x_i^h.

$$\beta_i = \alpha^j \frac{\partial f^j}{\partial y_i^j} \quad \text{for all i,j} \qquad [3]$$

$$\beta_i = \lambda^h \frac{\partial U^h}{\partial x_i^h} \quad \text{for all i,h} \qquad [4]$$

Plus

$$\beta_z = \alpha^j \frac{\partial f^j}{\partial z^j} \qquad [3a]$$

$$\beta_z = \sum_h \lambda^h \frac{\partial U^h}{\partial z} \qquad [4a]$$

Divide [4a] by any β_i :

$$\frac{\beta_z}{\beta_i} = \sum_h \frac{\lambda^h \partial U^h / \partial z}{\beta_i}$$

Make H substitutions using many FOC's [4]:

$$\frac{\beta_z}{\beta_i} = \sum_h \frac{\lambda^h \partial U^h / \partial z}{\lambda^h \partial U^h / \partial x_i^h} = \sum_h \frac{\partial U^h / \partial z}{\partial U^h / \partial x_i^h} = \sum_h MRS.$$

Using [3] & [3a], $\quad MRT = \frac{\beta_z}{\beta_i}$

so we get the Samuelson condition

$$MRT = \sum_h MRS.$$

[3], [3a], [4], & [4a] have more precision than the Samuelson condition so they are better PO conditions for our theory, but it's comforting to see how everything ties together.

Markets

Firms: $\quad L^j = py^j + p_z z^j - \delta^j f^j\left(y^j, z^j\right)$

$\Rightarrow$ [1] (not repeated here) and

$$p_z = \delta^j \frac{\partial f^j}{\partial z^j} \qquad [1a]$$

Consumers: $\quad L^h = U^h\left(x^h, z\right) + \gamma^h\left(p_z \omega_z^h + p\omega^h + \theta^h \pi - p_z z^h - px^h\right)$

↑ amount of z that h <u>chooses</u> to pay for

$\Rightarrow$ [2] (not repeated here) and

$$\frac{\partial U^h}{\partial z}\frac{dz}{dz^h} - \gamma^h p_z \le 0 \text{ and } \left(\frac{\partial U^h}{\partial z}\frac{dz}{dz^h} - \gamma^h p_z\right)\cdot z^h = 0 \qquad [2a]$$

For economies of some size, what can we expect of $\frac{dz}{dz^h}$?

If it $\to 0$, [2a] becomes

$$-\gamma^h p_z \le 0 \text{ and } -\gamma^h p_z z^h = 0$$

Hence, consumers choose $z^h = 0$ implying that there will not be any funds dedicated to z production.

Most importantly, [2a] does not align with [4a], so we have a market failure.

A fix, known as Lindahl pricing, removes z^h from the individual's decision calculus – forcing everyone to pay a share indicated by their valuations: *personalized prices*.

That is, charge everyone based on their $\frac{\partial U^h}{\partial z}$ so that total z revenue adds up to z costs.

Lindahl Equilibrium

[1], [1a], [2] unchanged

Consumer's new problem is

$$L^h = U^h\left(x^h, z\right) + \gamma^h\left(p_z\omega_z^h + p\omega^h + \theta^h\pi - p_z^h z - px^h\right)$$

[We'll want to choose all the p_z^h so that $\sum_h p_z^h = p_z$; so that total revenue equals costs.]

$$\frac{\partial U^h}{\partial z} = \gamma^h p_z^h \qquad \forall h$$

$$\frac{1}{\gamma^h}\frac{\partial U^h}{\partial z} = p_z^h \qquad \forall h$$

$$\sum_h \frac{1}{\gamma^h}\frac{\partial U^h}{\partial z} = \sum_h p_z^h$$

$$\sum_h \frac{1}{\gamma^h}\frac{\partial U^h}{\partial z} = p_z \qquad \text{[LE 2a]}$$

The latest result has the resemblance to the Pareto condition [4a] we seek. Hence, properly selected Lindahl (personal) prices for the nonrival good have the ability to achieve Pareto optimality.

Hence, nonrivalness by itself need not foil the achievement of economic efficiency. We do need to be able to price the good, however. If we cannot effectively exclude nonpaying consumers, then Lindahl pricing is not an available policy solution.