Handout 5
Rent for the Firm

A. Welfare analysis for the production sector is much simpler than for consumption because of the absence of income effects (assuming that risk is not a concern).

B. Using the same notation that we have employed before, we can define profit, quasi-rent (R), and producer surplus (P) in a situation where some factors of production are fixed.

$$\pi = p \cdot y - TFC; \qquad\qquad R = P = p \cdot y.$$

ΔR is the desired measure of welfare change caused by a change in price. If price changes in a way that does not cause the firm to cease production, then $\Delta R = \Delta\pi$ and $\Delta\pi$ is an okay measure. If, however, the price change causes a halt in production, $\Delta R \neq \Delta\pi$, because the firm not only loses its profits – it still incurs fixed costs. ΔR remains a correct measure.

C. Assuming that technology is captured by $f(y) \leq 0$, profit-maximization conditions are, as before,

$$p_n = \delta \cdot \frac{\partial f}{\partial y_n} \quad \text{for all n.} \qquad [1]$$

These conditions can be used to determine the firm's optimal excess supply functions: $y_n^* = y_n^*(p)$.

So, $\pi = p \cdot y^*(p) - TFC$ and $R(p) = p \cdot y^*(p)$.

Taking the derivative of R with respect to an arbitrary price,

$$\frac{\partial R}{\partial p_n} = y_n^*(p) + p \cdot D_n y^*(p)$$

where D_n is the $N \to N$ differential operator.

Equivalently, $\dfrac{\partial R}{\partial p_n} = y_n^*(p) + \sum_i p_i \dfrac{\partial y_i^*}{\partial p_n}$.

The final term of this expression is equal to zero according to the following proof. Differentiating $f(y) = 0$ totally, we obtain

$$\sum_i \left(\frac{\partial f}{\partial y_i}\right) \cdot \left(\frac{\partial y_i}{\partial p_n}\right) = 0.$$

Substituting from [1],

$$\sum_i \left(\frac{p_i}{\delta} \right) \cdot \left(\frac{\partial y_i}{\partial p_n} \right) = 0,$$

or

$$\delta^{-1} \sum_i p_i \frac{\partial y_i}{\partial p_n} = 0$$

Hence, $\frac{\partial R}{\partial p_n} = y_n^*(p)$. [Hotelling's Lemma] (2)

D. Suppose the initial price vector is p^0 and the subsequent price vector is p^1. Then, (no fixed costs here)

$$\Delta R = R\left(p^1\right) - R\left(p^0\right) = \pi^1 - \pi^0. \tag{3}$$

Equation [3] can be used to calculate the change in quasi-rent if you have complete knowledge on how prices affect profits. Usually, however, this information cannot be directly observed, and an alternative method is popular:

E.
$$\begin{aligned} \Delta R = R\left(p^1\right) - R\left(p^0\right) &= \int_L dR \\ &= \int_L \left(\sum_n y_n^*\left(p\right) dp_n \right) \\ &= \sum_n \int_{p_n^0}^{p_n^1} y_n^*\left(p\right) dp_n. \end{aligned} \tag{4}$$

Equation [4] has chosen a particular path of integration (L), but the path is unimportant because of path independence. "Path" refers to a selected sequence of price changes.

F. ΔR for an output price change (p_n)

It is important to observe a crucial assumption in all the following formulae of this handout. It is assumed that no other prices change in response to the analyzed price change(s). That is, a partial equilibrium situation is assumed. Some authors handle this by assuming that all other supplies and demands are perfectly elastic.

1. Measured in the output market (Figure 1.1) where p_n^{min} indicates p_n at which marginal cost curve intersects average variable costs.

$$R^0 = \int_{p_n^{min}}^{p_n^0} y_n^* dp_n = a + b$$

$$R^1 = \int_{p_n^{min}}^{p_n^1} y_n^* dp_n = b$$

$$\Delta R = \int_{p_n^0}^{p_n^1} y_n^* dp_n$$

$$= R^1 - R^0 = -a$$

2. Measured in an input (y_i) market (Figure 1.2)

   Full measurement of an output price change in an input market requires that the input be *essential* (Just, Hueth, Schmitz, p. 63). An input is essential if zero input usage implies zero output.

$$R^0 = \int_{\bar{p}_i}^{p_i^{max}} \left(-y_i^*\left(p^0\right)\right) dp_i = a + b$$

$$R^1 = \int_{\bar{p}_i}^{p_i^{max}} \left(-y_i^*\left(p^1\right)\right) dp_i = b$$

$$\Delta R = \int_{\bar{p}_i}^{p_i^{max}} \left[-y_i^*\left(p^1\right) - \left(-y_i^*\left(p^0\right)\right)\right] dp_i$$

$$= R^1 - R^0 = -a$$

G. ΔR for an input price change (p_i)

1. Measured in the output market (Figure 2.1)

$$R^0 = \int_{p_n^{min}}^{\bar{p}_n} y_n^*(p^0) dp_n = a$$

$$R^1 = \int_{p_n^{min}}^{\bar{p}_n} y_n^*(p^1) dp_n = a + b$$

$$\Delta R = \int_{p_n^{min}}^{\bar{p}_n} \left[y_n^*(p^1) - y_n^*(p^0)\right] dp_n$$

$$= R^1 - R^0 = b$$

2. Measured in the input market (Figure 2.2)

$$R^0 = \int_{p_i^0}^{p_i^{max}} (-y_i^*)dp_i = a$$

$$R^1 = \int_{p_i^1}^{p_i^{max}} (-y_i^*)dp_i = a + b$$

$$\Delta R = \int_{p_i^1}^{p_i^0} (-y_i^*)dp_i$$

$$= R^1 - R^0 = b$$

H. ΔR for multiple price changes

1. Sequential measurement is always possible but must be performed carefully.
2. Quasi-rent can be measured in any market (as always). Referring to Figure 3 as an example for an output market:

$$R^0 = \int_{p_n^{min}}^{p_n^0} y_n^*\left(p^0\right)dp_n = a + b$$

$$R^1 = \int_{p_n^{min}}^{p_n^1} y_n^*\left(p^1\right)dp_n = a + d$$

$$\Delta R = R^1 - R^0 = d - b$$

References

Just, E. Richard, Darrell L. Hueth, and Andrew Schmitz. *The Welfare Economics of Public Policy*. Cheltenham, UK: Edward Elgar, 2004.

Mishan, E.J. *Introduction to Normative Economics*. New York: Oxford University Press, 1981.

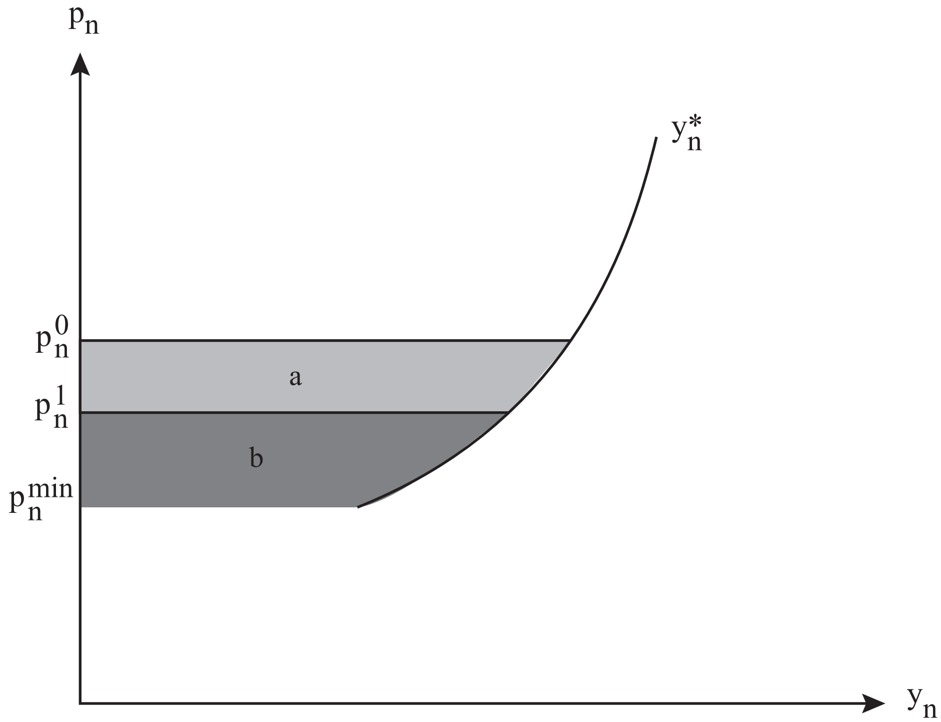

Figure 1.1

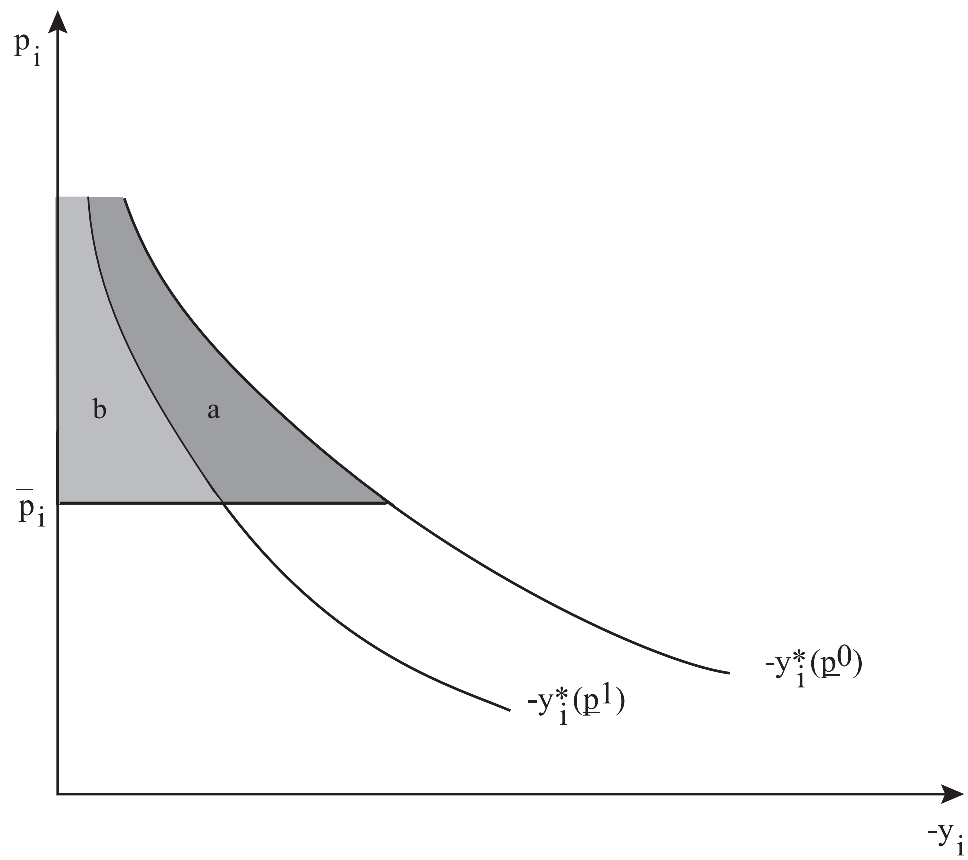

Figure 1.2

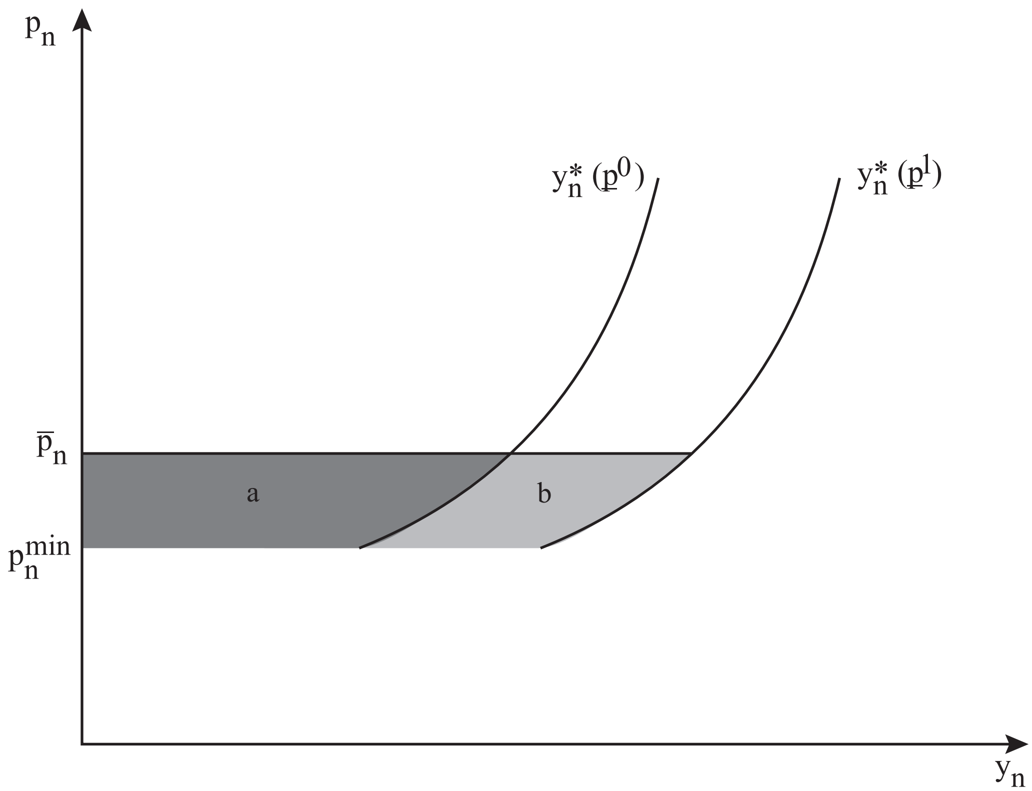

Figure 2.1

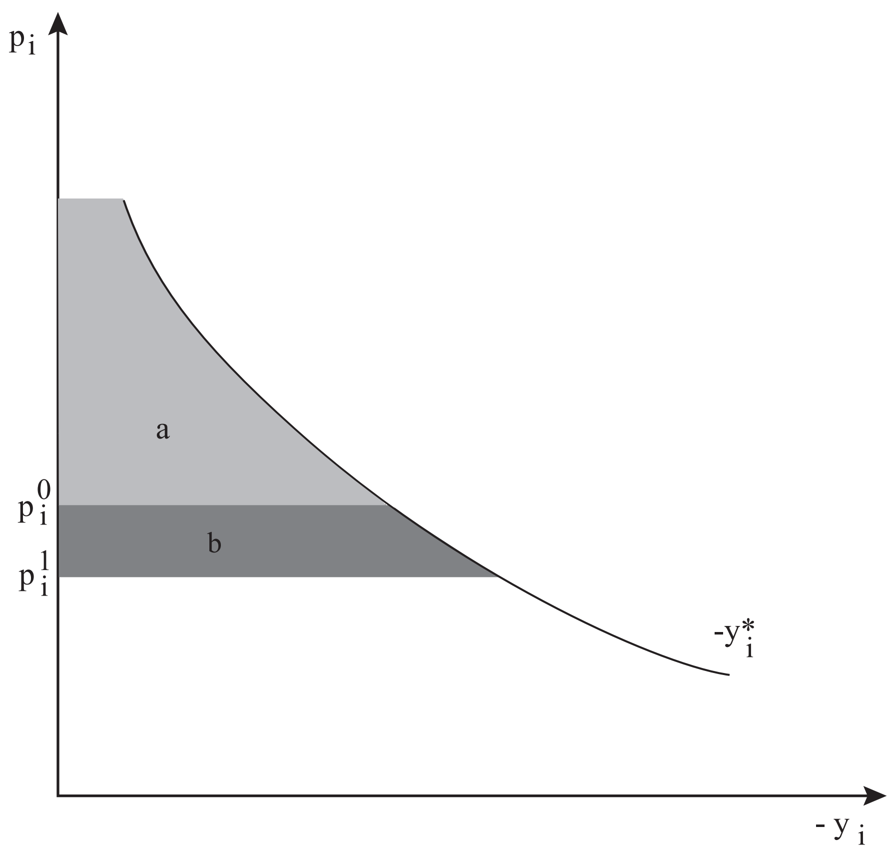

Figure 2.2

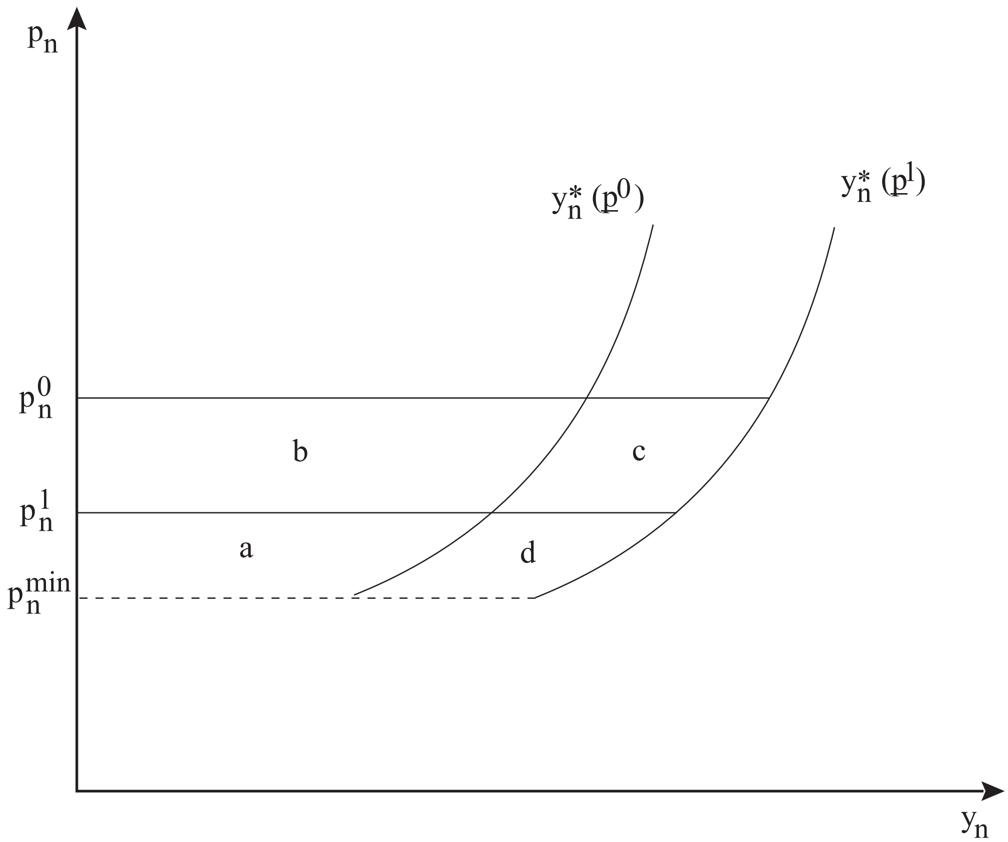

Figure 3