Handout 6

Welfare Measures for the Consumer

A. For a given project/policy impact upon a given individual, we require a measure that fully captures the effects of the change. It is for this reason that we are inevitably interested in measures derived from compensated (Hicksian) demand functions. See Figure 1 for the Hicksian demands and ordinary (Marshallian) demand corresponding to an increase in price. Because of the income effect, a consumer's utility changes as you move along his/her Marshallian demand curve. But we wish to determine a monetary measure which, if used as compensation to the consumer, leaves the consumer's utility unchanged (from some "base" utility level) after the policy takes place. Hicksian measures are sometimes called *exact* welfare measures because of this.

B. The simplistic, two-base view is that we wish to calculate what amount of income change precisely offsets either

1. the project/policy impact (using the initial situation as the base) or
2. the absence of the project/policy impact (using the proposed subsequent situation as the base).

C. For each alternative base situation (i.e., distribution of income and the resultant utilities), you will presumably obtain a different measure of the welfare change. This point is fundamental.

Alternative Welfare Measures	
Compensating Variation (CV) Compensating Surplus (CS)	Kaldor Compensation Tests "status quo ante"
Equivalent Variation (EV) Equivalent Surplus (ES)	Hicks Compensation Tests "status quo post"

These measures refer to but two of an infinite number of alternative income distributions. Note that the Kaldor test, the Hicks test, and the use of any other status quo position can be used to calculate a Hicksian welfare measure.

D. *Variation* measures are appropriate for price changes – where the consumer has the opportunity to adjust his/her consumption bundle given the price change. *Surplus* measures are appropriate for quantity changes – which the consumer must take as given.

E. Compensating Variation

1. Given the indifference map in Figure 2, point A depicts the initial consumption bundle. A proposed policy will decrease the price of x_1 relative to all other goods.
2. For *compensating* measures, the initial situation defines U_1 as the basis for comparison, and we must use a *variation* measure because we are dealing with a price change.
3. The CV question is then: With the subsequent price vector, what lump-sum amount (+ or –) is necessary to bring the individual back to U_1?

a. The answer to this question is the CV (as illustrated in Figure 2).

b. In response to this hypothetical lump-sum transfer, the individual would presumably adjust his/her consumption bundle to that identified as point C.

4. In order to mathematically define CV, we require the concept of an expenditure function. $e(p, \bar{U})$ is the solution to

$$\text{minimize } \underline{p} \cdot \underline{x} \text{ subject to } U(\underline{x}) \geq \bar{U}.$$

This function identifies the minimum expenditure needed to obtain a given level of utility at given prices.

5. When all prices but p_1 are fixed (as in Figure 2), it is easily seen that

$$\begin{aligned} CV &= e(p_1'', U_2) - e(p_1'', U_1) \\ &= e(p_1', U_1) - e(p_1'', U_1). \end{aligned}$$

6. As a means to derive an equivalent expression for CV, recall that the compensated (Hicksian) demand function is given by

$$h_n(\underline{p}, \bar{U}) = \partial e / \partial p_n.$$

7. Hence,

$$\begin{aligned} \int_{p_1''}^{p_1'} h_1(\rho_1, U_1) d\rho_1 &= e(\rho_1, U_1) \Big|_{p''}^{p'} \\ &= e(\rho_1', U_1) - e(\rho_1'', U_1) = CV. \end{aligned}$$

8. Under certain conditions compensated demand functions may be well approximated by ordinary (Marshallian) demand functions. Hence, the change in consumer surplus (ΔS) may approximate CV:

$$CV \overset{?}{\cong} \Delta S = \int_{p_1''}^{p_1'} x_1(\rho_1, M) d\rho_1 ..$$

F. Equivalent Variation

1. The proposed subsequent situation defines U_2 as the basis for this welfare measure (see Figure 3).

2. The EV question is: With the initial price vector, what income change is required to bring the individual to U_2?

3. $EV = e(p_1', U_2) - e(p_1', U_1)$

$$= e\left(p_1', U_2\right) - e\left(p_1'', U_2\right)$$

$$= \int_{p_1''}^{p_1'} h_i\left(\rho_1, U_2\right) d\rho_1$$

G. Comparing CV, EV, and ΔS

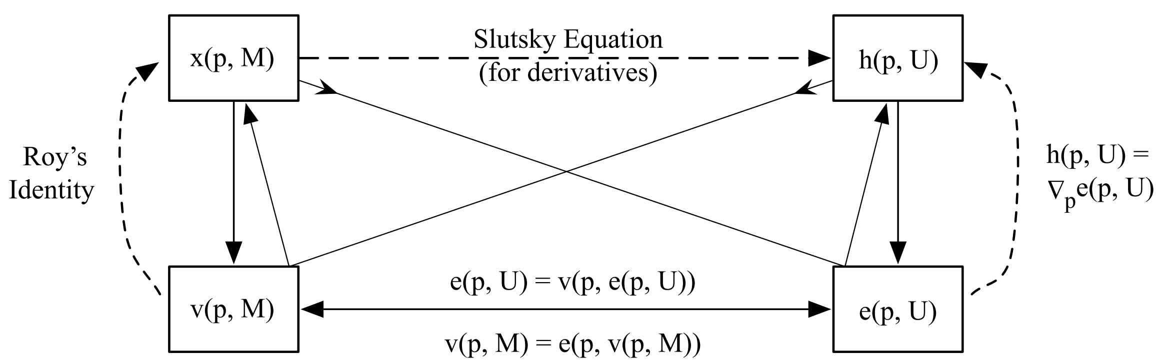
1. For price decreases, $0 < CV \leq \Delta S \leq EV$.
2. For price increases, $CV \leq \Delta S \leq EV < 0$. Verify this by inspecting Figure 1.
3. Due to the Slutsky equation,

$$\frac{\partial x_n}{\partial p_n} = \frac{\partial h_n}{\partial p_n} - x_n\left(\frac{\partial x_n}{\partial M}\right),$$

it can be easily seen that $CV = \Delta S = EV$ when there is no income effect.

H. Calculation options from preference-embedding functions

Due to duality findings, there is an array of paths for calculating consumer welfare measures, depending on what information is readily available. From Mas-Colell, Whinston, and Green (1995, p. 75):



If v(p,M) is known and U(x) is desired, convert v(p,M) to the form v(q) where $q = \left(\frac{p_1}{M}, \frac{p_2}{M}, \ldots\right)$ and to get U(x) solve $\underset{q}{\text{Min}}\, v(q)$ s.t. $qx \leq 1$. Alternatively and equivalently, use Roy's identify to acquire the demand functions, and rearrange these so they can be substituted into the indirect utility function to remove the $\frac{M}{p_i}$.

I. Compensating Surplus

1. For some situations, as with the government provision of a public or semi-public good or with a legal restriction on the use of some good, compensating variation is an inappropriate welfare measure because it presumes that, following the lump-sum

transfer of CV, the consumer will be able to optimally adjust his/her consumption bundle (from B to C in Figure 2). This adjustment is not always possible, because the individual's consumption of x_1 may be fixed at x_1'' by the proposed policy.

2. In this case the appropriate compensating measure is the compensating surplus depicted by line segment BF in Figure 4. [The depiction of Figure 4 is used to assist understanding, but it does not offer a computational means because we normally cannot know indifference curves.]

3. If the compensated demand function is given by $x_1 = h_1(p_1, \bar{U})$, then we can use the inverse compensated demand function, $p_1 = \tilde{h}_1(x_1, \bar{U})$, to mathematically describe CS. [Even though the good x_1 is not being allocated by pricing, pricing must usually be imagined in order to obtain any demand function and its inverse.]

   a. Assuming the initial price for x_1 remains at p_0 both before and after the policy change (where p_0 is too low to be rationing and may be zero) and quantity changes from x_1' to x_1'':

$$CS = \int_{x_1'}^{x_1''} \left[\tilde{h}_1(x_1, U_1) - p_0\right] dx_1$$

   Illustrating this calculation requires an obvious reconstruction of Figure 1 in which we integrate over a quantity change <u>under</u> $h_1(p_1,U_1)$.

   b. If price (not a fixed fee) also changes, then a slightly more complicated formula is necessary, but as long as the price remains too low to be rationing, welfare measurement is simple. Note that the p_0 term in the prior integral for CS results in the deduction of $\Delta x \bullet p_0$ from the expression. Extending this reasoning when p changes will modify the $-\Delta x \bullet p_0$ term to $-(p_1 x_1 - p_0 x_0)$.

   c. In many real-world settings there may be a fixed fee (rather than a price) to be paid in order to access the good being valued. Examples include entrance fees and license fees, as well as other fees which are invariant with the amount the consumer chooses to consume. Obviously, if the policy proposal includes a change in fees, then the Δfee is also pertinent as an additional welfare measure.

J. Equivalent Surplus

1. Using the post project/policy utility level of U_2, the ES measure is entirely analogous to CS. ES is graphically depicted by the length of segment AE in Fig. 5.

$$ES = \int_{x_1'}^{x_1''} \left[\tilde{h}_1(x_1, U_2) - p_0\right] dx_1$$

K. ΔS for a Quantity Change

1. Similar to the above cases of CS and ES, the computation of ΔS for an imposed quantity change requires integration under the inverted Marshallian demand function (as opposed to integration beside the Marshallian demand function for a price change as in section E8 above). Writing the appropriate integral is left as an exercise for you:

$$\Delta S =$$

L. Comparing CS, ES, and ΔS

1. For quantity increases in a beneficial good, $0 < CS \leq \Delta S \leq ES$.
2. For quantity decreases in a beneficial good, $CS \leq \Delta S \leq ES < 0$.

M. Willig-type Results

1. Comparisons among the Hicksian and Marshallian welfare measures offered above in sections G and K show ΔS to be bounded on either side by the available Hicksian measures. A very important question for applied work is "How well does ΔS approximate the exact welfare measures under typical circumstances?" This query is motivated by the fact that the Marshallian measure is far easier to obtain because market data readily permits estimation of ordinary demand(s). Led by Willig's *AER* article in 1976, the literature has addressed this question in the context of single price changes (Willig), multiple price changes including wages (Just, Hueth, and Schmitz), and quantity changes (Randall and Stoll). The general finding is that the approximation is good in most cases. The accuracy of the approximation generally depends on (a) income elasticity(ies) and (b) the size of ΔS relative to income. Increases in either of these factors decreases accuracy. Quantitative guidelines for estimating the degree of accuracy are provided by this literature should you choose to pursue ΔS and wish to know how wrong you might be.
2. In some cases Hicksian measures can also be obtained using market data, but that is the subject of yet another handout.

References

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Randall, Alan and John R. Stoll. "Consumer's Surplus in Commodity Space." *American Economic Review* 70 (June 1980): 449-55.

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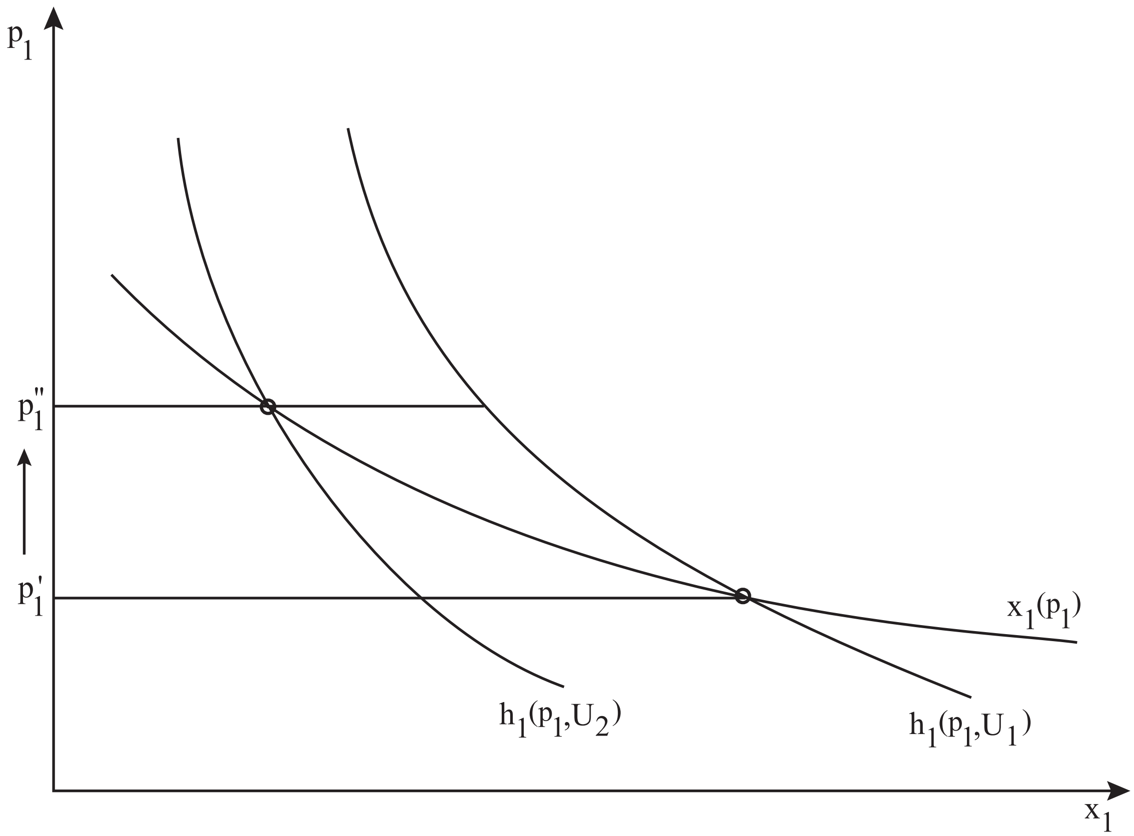

Figure 1

Variations (for a price decrease)

Point A represents the initial situation with p_1'. Point B is the subsequent situation with p_1''.

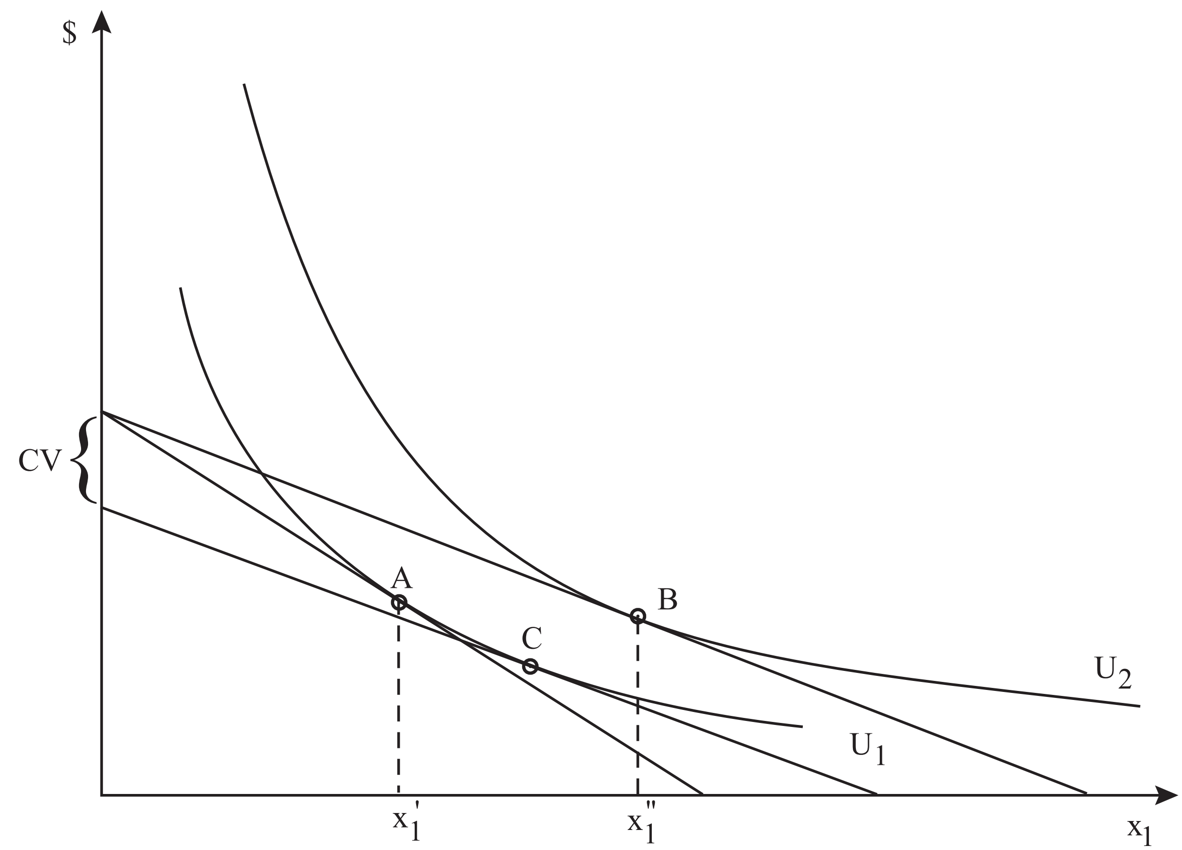

Figure 2

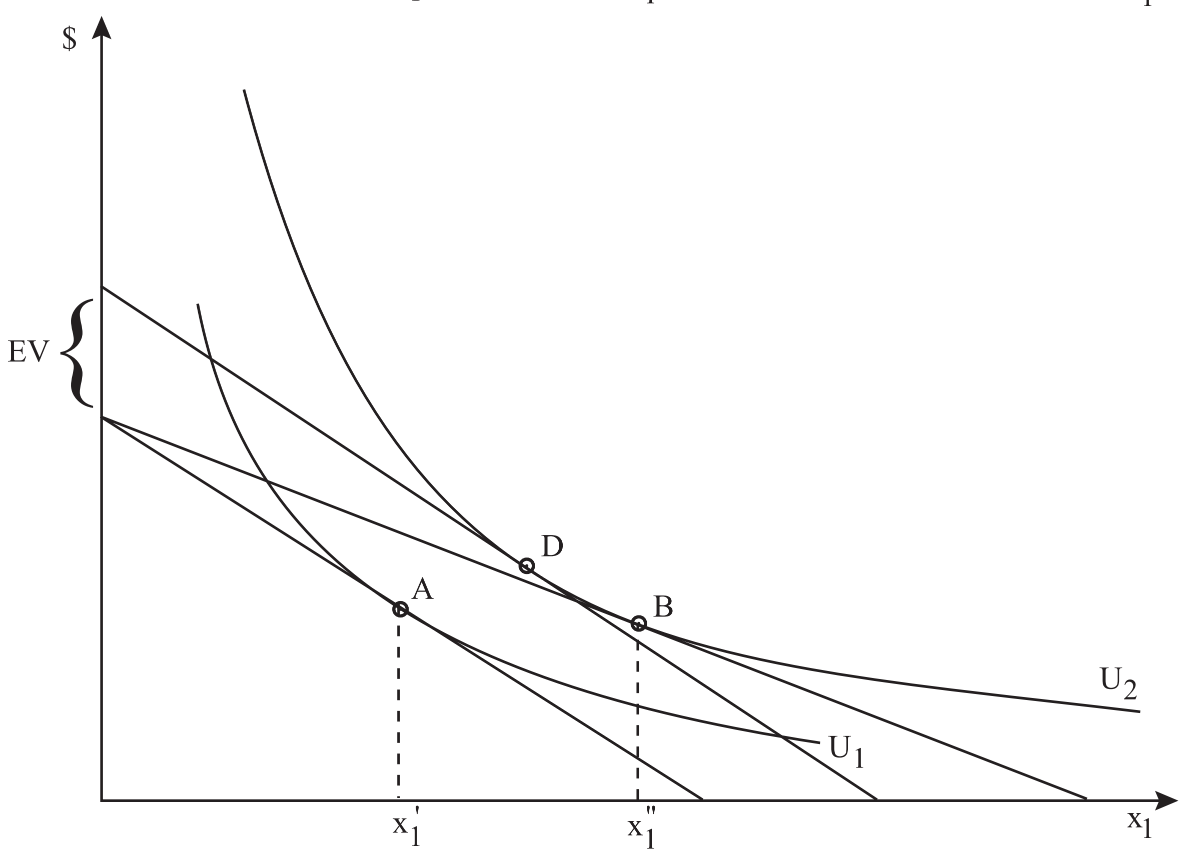

Figure 3

Surpluses (for a quantity increase)

Point A designates the initial situation with x_1'. Point B designates the subsequent situation with x_1''.

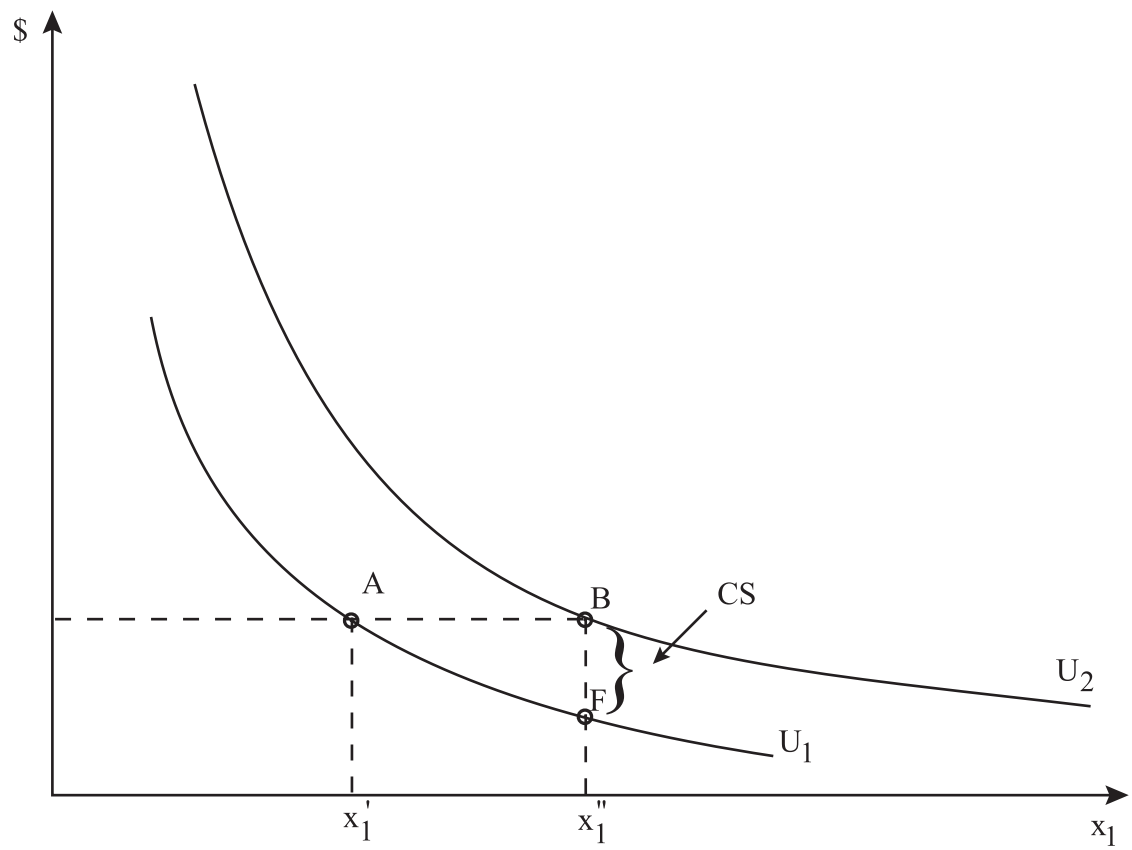

Figure 4

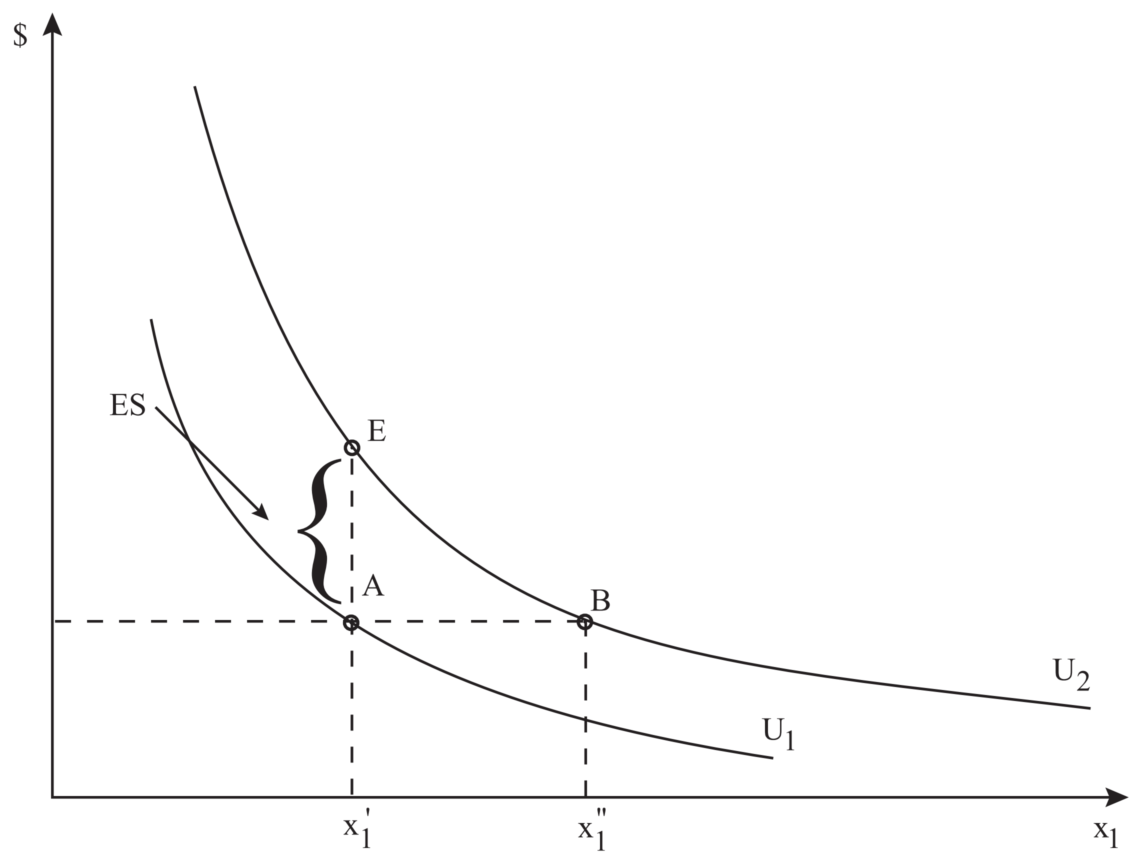

Figure 5

