Handout 7

Variances and Confidence Intervals for Welfare Measures (and Other Things)

Welfare measures are functions of supply and demand parameters – things which are not tightly known in real-world circumstances. Because these parameters are often estimated econometrically, they are random variables and anything computed using them are random variables as well. Rather than simply reporting the expected value of a welfare measure in these situations, it can be informative to report the welfare measure's variance or its confidence interval. Alston and Larson's paper[1] makes use of the following common technique (Taylor series approximation) for accomplishing this.

1. You've statistically estimated some relation (e.g. demand, supply, or production function) using some observed variables, and your parameter estimates are $\breve{\underline{\delta}}$. The true $\underline{\delta}$ is unknowable.

2. You wish to compute something with this $\breve{\underline{\delta}}$. Let this something be Y, and suppose it is a nonlinear function of $\underline{\delta}$ and some other known variables, $\underline{x}$:

$$Y = h\left(\underline{\delta};\underline{x}\right).$$

3. But you don't know $\underline{\delta}$. You know only the elements of $\breve{\underline{\delta}}$ which are random. Therefore, you can only use h to compute an estimate of Y which we'll call $\breve{Y}$.

$$\breve{Y} = h\left(\breve{\underline{\delta}};\underline{x}\right).$$

4. What is the variance of $\breve{Y}$?

5. Taking a Taylor series expansion of h around the unknown vector $\underline{\delta}$,

$$\breve{Y} = h\left(\underline{\delta};\underline{x}\right) + h'\left(\breve{\underline{\delta}};\underline{x}\right)\cdot\left(\breve{\underline{\delta}} - \delta\right) + \text{higher order terms}$$

   where $h\left(\breve{\delta};\underline{x}\right)$ is the true and unknown Y, and $h'\left(\breve{\underline{\delta}};\underline{x}\right)$ is a vector of partial derivatives of h with respect to the elements of $\underline{\delta}$. The higher order terms, which tend towards zero, are dropped in this approach to obtain a workable linear approximation.

6. Rearranging,

$$\breve{Y} - Y \cong h'\left(\breve{\underline{\delta}};\underline{x}\right)\cdot\left(\breve{\underline{\delta}} - \delta\right).$$

7. Therefore, an estimated variance for $\breve{Y}$ is:

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[1] Alston, Julian M. and Douglas M. Larson. "Hicksian vs. Marshallian Welfare Measures: Why Do We Do What We Do?" *American Journal of Agricultural Economics* 76 (August 1993): 764-9.

$$s_{\breve{Y}}^2 \cong \left(h'\left(\underline{\breve{\delta}};\underline{x}\right)\right) \cdot \mathrm{Var}\left(\underline{\breve{\delta}}\right) \cdot \left(h'\left(\underline{\breve{\delta}};\underline{x}\right)\right)^{\text{transpose}}$$

where $\mathrm{Var}\left(\underline{\breve{\delta}}\right)$ is the square matrix of estimator covariances available from your earlier econometric work.

8. Now, having an estimated variance you can report it, and you can use it to obtain a confidence interval about $\breve{Y}$. The most appropriate confidence interval is given by

$$\breve{Y} \pm t_{(T-1,\alpha/2)}\left(\frac{s_{\breve{Y}}}{\sqrt{T}}\right)$$

where the value $t_{(T-1,\alpha/2)}$ refers to the element of the *t*-distribution for T degrees of freedom and a 100(1-α)% confidence interval.