2.12 Exercises

1. Build a spreadsheet in which you sample from the production function $y = 3x^2 - 0.2x^3$ from $x = 0$ to $x = 12$ in increments of $\Delta x = 0.5$ or smaller. Place the following information in the parenthetically numbered columns of your spreadsheet: (1) x, (2) y, (3) Δx, (4) Δy, (5) $\Delta y/\Delta x$. Place the production function formula in each entry of column 2. Use formulas elsewhere when you can. The entries in column 3 will all be the same. The entries in column 4 will be obtained as $y_{row} - y_{row-1}$ computed using column 2 information. The results contained in column 5 are your estimates of marginal product, and these estimates get progressively more accurate as Δx becomes smaller. Plot columns 2 and 5 against column 1 to reproduce the upper panel of figure 2.3. Assume a specific fixed price for y (anything will do), and create a sixth column containing $\text{price}\bullet\Delta y/\Delta x$. Graph this new column against column 1 to produce a demand function (as in figure 2.4).

2. Suppose that irrigated corn can be produced using water and nitrogen fertilizer according to the following statistically estimated production function (using data from field experiments):

$$\text{corn} = -10586 + 688.36W + 36.421N - 10.039W^2 - 0.0772N^2 + 0.4133WN$$

where corn and nitrogen units are pounds per acre, and water units are acre-inches (Hexem and Heady 1978, 78-81; many similar functions are described in their book). Using a spreadsheet or other program and the following assumptions, generate each panel of figures 2.2 and 2.3. Assumptions: $N = 240$, $0 \le W \le 50$, corn price = 4 cents/lb., and marginal cost of water = \$5/acre-inch. What's the profit-maximizing amount of water to apply? (Don't be too surprised if your functions have a different shape than those in the text. Shape is dictated by functional form, and Hexem and Heady use a quadratic form for this production function.)

3. Given only the definitional information for average and marginal costs ($AC = C/w$ and $MC = dC/dw$), prove that the average cost curve is intersected by the marginal cost curve at the minimum of average costs. (Hint: given the definition of average costs, minimize it.)

4. Make up an original (w, p) demand point (choose numbers). Select a believable value for demand elasticity. Apply the point expansion method to precisely identify a demand function corresponding to your assumptions. Clearly state the resulting demand function. Suppose that marginal costs are given by $MC = w/10$. Determine the applicable MNB function.

5. How should we interpret a situation in which aggregate MNB of figure 2.10 does not intersect $w = W$ or, equivalently, the curves of figure 2.11 do not intersect at a positive price?

6. Two distinct agents have the following marginal net benefit functions for retail water:

$$MNB_1 = 300 - 5w_1 \quad \text{and} \quad MNB_2 = 200 - 2w_2 .$$

Seventy-six units of water are available for allocation between these agents.

a. What is the aggregately efficient allocation of water? What are the consequent marginal net benefits and net benefits for 1 and 2?

b. Quantitatively describe the neutrally efficient allocation of water with an equation relating w_1 and w_2. As a consequence of this efficient range of allocations, quantitatively describe the relation between MNB_1 and MNB_2. Now describe this efficiency relation between MNB_1 and MNB_2 parametrically, writing both as a function of a single variable, w_1.

c. For arbitrary water allocations w_1 and w_2, compute the relationships $NB_1(w_1)$ and $NB_2(w_2)$. Assuming neutral efficiency, rewrite $NB_2(w_2)$ as a function of w_1. Given this parametric specification for relating NB_1 and NB_2 under neutral efficiency, determine if the aggregately efficient allocation is also neutrally efficient.

7. Monthly water use in the City of Utility is 32 million gallons, and the average household is paying \$3 per thousand gallons. If the price elasticity of demand is –0.75, what is a good estimate of the aggregate water demand function in Utility? What units do the variables of your function utilize? Suppose that the rate is set at \$3 because each and every 1000 gallons delivered to a household costs Utility exactly \$3 to process and deliver, excluding any costs of natural water. What is the town's marginal net benefit function?

8. Cornrus Irrigation District (CID) charged its members \$20 per acre-foot last year and was then asked to deliver 40,000 acre-feet to these members. Statistical studies of Cornrus and its neighboring irrigation districts place the price elasticity of water demand at –1. Provide and document two justifiable estimates of the price CID should select if it wants its members to demand 50,000 acre-feet this year.

9. Businesses A and B have the following demands for retail (processed) water: $w_A = 1200 - 2p$ and $w_B = 600 - p$ where w's represent thousands of gallons and p is \$/thousand gallons. The utility supplying both businesses has marginal processing costs of \$10 per thousand gallons, and it can deliver 960,000 gallons total (no more) to A and B combined. How should this water be divided? Is there a surplus of water? Provide a quantitative solution.

10. You are a manager within a water-using business that expects no change in future demand for its products. Your water is self-supplied using a permitted water source from which you pump and treat water. It costs you a constant \$25 per unit of water for every unit used. Company water demand is $w = (750/p)^2$. The company owns 1200 units of transferrable water rights that it cannot exceed when pumping. The company's owner asks the following: "I want to lease one-third of our water rights to my daughter's company. It's important that my company break even on this lease – experiencing neither a loss nor gain. How much money I should ask her to pay? Explain all this stuff to me using whatever tools you have."

11. Every household in a 100-household town has two demands for water: in-home consumption demand (w_h) and at-the-public-park recreation demand (w_r). The park's central feature is a large lake. The average household's demand information is contained in these two expressions:

$$mb_h = \left(\frac{1}{w_h}\right)^{1/3} \quad \text{and} \quad mb_r = \left(\frac{16}{w_r}\right)^{1/4}.$$

What is the town's total demand for water? Explain and justify your procedure.

12. One of the basin's water users cheated last year by taking more water than she was entitled to. She has been found guilty and must pay damages. Last year's demand for natural water (in terms of \$/unit) by all other permitted owners was $mb=1000 - 40w^{0.5}$. They were allowed to take 676 units of water according to their total permits. Because of the cheater, they could only take 576 units of water. What is your assessment of the damages? What does your analysis assume about how well all the other permit holders allocated their limited supply?

13. Under average weather conditions, total water use in a summer month is 60 million gallons when consumers expect to be charged \$8/1000 gallons. The price elasticity of demand is –0.4. \$8/1000 gallons is also the utility's marginal cost of supplying water.

If drought implies that the utility can only supply 90% of summer-month quantity demanded at the normal price (assuming drought demand is the same as average-weather demand), what is the "loss due to drought" for a summer month? Illustrate with a graph and compute.