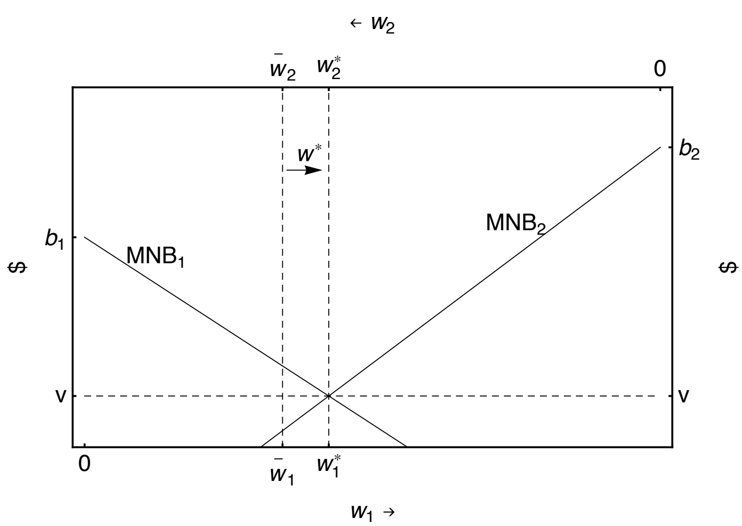
graphing format

```
thinn = AbsoluteThickness[.5];
medum = AbsoluteThickness[1.];
thick = AbsoluteThickness[1.5];
black = GrayLevel[0];
BGray = GrayLevel[0.3];
WGray = GrayLevel[0.6];
LGray = GrayLevel[0.8];
SetOptions[Plot, PlotStyle → {{thinn, Black}, {thinn, Black}, {thinn, Black}},
  PlotPoints → 40, ImageSize → 360,
  FrameStyle -> medum, AxesStyle -> medum,
  BaseStyle -> {FontFamily -> "Helvetica", FontSlant → Plain, FontSize → 12}];
SetOptions[ListPlot, AxesStyle -> medum, PlotStyle → medum, ImageSize → 384,
  BaseStyle -> {FontFamily -> "Helvetica", FontSlant -> "Plain", FontSize → 12}];
SetOptions[ParametricPlot, PlotStyle →
   {{thinn, Black}, {thinn, Black}, {thinn, Black}}, PlotPoints → 40,
  FrameStyle -> medum, AxesStyle -> medum, PlotStyle → medum,
  BaseStyle -> {FontFamily -> "Helvetica", FontSlant -> "Plain", FontSize → 12}];
SetOptions[Graphics, BaseStyle ->
   {FontFamily -> "Helvetica", FontSlant -> "Plain", FontSize → 12}];
```

Figure 8.1

```
{b1, b2, m1, m2, w̄1, w̄2} = {7., 10., 0.78, .9, 5.5, 10.5};
MNB1 = b1 - m1 * w;
MNB2 = b2 - m2 * w;
totalw = w̄1 + w̄2;
flipMNB2 = MNB2 /. w → (totalw - w);
Solve[MNB1 == flipMNB2, w];
w1* = w /. %[[1]];
v = MNB1 /. w → %;
w2* = w̄1 + w̄2 - w1*;
w* = w̄2 - w2*;
pl81a = Plot[{MNB1, flipMNB2}, {w, 0, totalw},
    FrameLabel → {"w1 →", "$", "← w2", "$"},
    PlotRange → {0, 1.2 * Max[b1, b2]},
    Frame → True,
    FrameTicks → {{{0, "0", {0.00625, 0.}, {black, medum}}, {w̄1, "w̄1", {0.00625, 0.},
         {black, medum}}, {w1*, "w1*", {0.00625, 0.}, {black, medum}}},
       {{v, "v", {0.00625, 0.}, {black, medum}}, {b1, "b1", {0.00625, 0.},
         {black, medum}}}, {{totalw, 0, {0.00625, 0.}, {black, medum}},
        {w̄1, "w̄2", {0.00625, 0.}, {black, medum}}, {w1*, "w2*", {0.00625, 0.},
         {black, medum}}}, {{v, "v", {0.00625, 0.}, {black, medum}},
        {b2, "b2", {0.00625, 0.}, {black, medum}}}}];
pl81 = Show[pl81a,
   Graphics[Text["MNB1", {2, 6.4}]],
   Graphics[Text["MNB2", {12, 7.5}]],
   Graphics[Text["w*", {6.25, 9.8}]],
   Graphics[{Arrowheads[0.03], Arrow[{{5.61, 9.24}, {6.61, 9.24}}]}],
   Graphics[{Dashing[{.01, .01}], thinn,
     Line[{{0, v}, {totalw, v}}], Line[{{w̄1, 0}, {w̄1, 1.2 * Max[b1, b2]}}],
     Line[{{w1*, 0}, {w1*, 1.2 * Max[b1, b2]}}]}], ImageSize → 384,
   AspectRatio →
    0.6]
```



Basic Theory check

$MNB_1 = b_1 - m_1 * w_1;$
$MNB_2 = b_2 - m_2 * w_2;$

$$\Delta R_1 = \int_{\overline{w}_1}^{\overline{w}_1 + w^*} MNB_1 \, d w_1$$

2.83959

$$\Delta R_2 = \int_{\overline{w}_2}^{\overline{w}_2 - w^*} MNB_2 \, d w_2$$

-1.45102

Figure 8.8

For better scale, I'll put region 1 (area of origin) on y-axis.

Decreasing returns to scale benefit functions for 2 regions

```
{a, b, c, d} = {5., 0.45, 5., 0.55};
W̄ = 40.;
B1 = a * w1 ^ b;
B2 = c * (W̄ - w1) ^ d;
topR = 1.2 * c * (W̄) ^ d;
topO = 1.2 * a * (W̄) ^ b;
pl88a = ParametricPlot[{B2, B1}, {w1, 0., W̄},
    PlotRange → {{0., topR}, {0., topO}},
    AxesLabel → {$Receipt, $Origin},
    Ticks → None];
```

Find point of max total benefits on the frontier

```
mb1 = D[B1, w1]
mb2 = D[B2, w1]
```

$$\frac{2.25}{w1^{0.55}}$$

$$-\frac{2.75}{(40. - w1)^{0.45}}$$

```
FindRoot[mb1 == -mb2, {w1, 1.}, MaxIterations → 35]
```

{w1 → 10.9356}

```
optw = w1 /. %;
A1 = B₁ /. w1 → optw;
A2 = B₂ /. w1 → optw;
A = {A2, A1}
```

{31.9021, 14.6706}

Find equation for tangent line at point A

We know slope (-1) and a point (A), so find intercept.

```
intercept = A1 + 1. * A2
```

46.5727

```
line = - B2 + intercept
```

46.5727 - B2

Get 2 suitable line segment endpoints for tangent line.

```
end1 = {A2 - 8., line /. B2 -> A2 - 8.};
end2 = {A2 + 9., line /. B2 -> A2 + 9.};
```

Final Plot

```
pl88 = Show[pl88a,
  Graphics[Text["A", {34., 15.}]],
  Graphics[Text["slope=-1", {26., 25.}]],
  Graphics[ {PointSize[0.018], Point[A]}],
  Graphics[{Dashing[{.01, .01}], medum, Line[{end1, end2}]}],
  ImageSize → 360,
  AspectRatio → 0.5]
```